Sophomore and Junior Assignments - Becker

Graphing Rational Functions

- What is a rational function?
  - A rational function is a function expressed as a ratio, or fraction.
  - For example; if we are given \( xy = 3 \) we can rearrange it to equal \( y = \frac{3}{x} \) by dividing the variable \( x \) on both sides. This is called inverse variation since we inverted the equation with the manipulation. We can graph this inverse variation. It will not create a straight line when you have a variable in the denominator. It creates a fraction that changes as you change the value of \( x \), if you plotted each value of \( x \) and the fraction it creates, it would create a curved line when you connect the dots.

  - \((1, 3); (2, \frac{3}{2}); (3, 1); (4, \frac{3}{4}); (5, \frac{3}{5}); (6, \frac{1}{2}); (7, \frac{3}{7}); (8, \frac{3}{8}); (9, \frac{1}{3})\)
  - Note the \( y \) coordinate; as you plug in larger values of \( x \) in the denominator the number gets smaller. All fractions are reduced.
  - The same would be true for all negative value of \( x \) also
  - With both positive and negative values being graphed, the graph is:

    \[
y = \frac{3}{x}
    \]

  - Remember: it is NOT possible to divide by the number 0 in ANY fraction. So wherever the denominator equals 0, we have what is called an asymptote. An asymptote is a vertical line through the graph at the value where the denominator equals zero, in this case that line is at \( x = 0 \)

  - In the example above; \( y = \frac{3}{x} \); if we take the denominator and set it equal to zero and solve the equation, we will find the \( x \) coordinate value of where the graph will have a vertical line. You can see in this example there is a line down the middle of the graph that both sides approach, but never touch

  - If we look at a different example; \( y = \frac{1}{x-2} \) and solve where the denominator equals zero: \( x - 2 = 0; x - 2 + 2 = 0 + 2 \) becomes \( x = 2 \). That means there will be a vertical line at the value of \( x = 2 \). The graph is shown below. Notice the vertical line at \( x = 2 \) and that the line approaches but never touches that value. You could create a table of \( x \) and \( y \) values to find each specific coordinate if you were asked by evaluating the expression.

    - The negative in the denominator causes a shift in the asymptote, with the asymptote at the value of \( x = 2 \). The horizontal asymptote is represented by \( y = 0 \)
Identify the asymptotes of each equation by solving where the denominator of each function equals zero.

1. \( f(x) = \frac{2}{x} \)
2. \( y = \frac{1}{x+2} \)

3. \( h(x) = \frac{3}{2x-4} \)

Identify the asymptotes of each graph on both \( x \) and \( y \) axis. For the vertical asymptote, write \( x = \) your answer. For the horizontal asymptote, write \( y = \) you answer. Remember, a horizontal line is graphed \( y = a \) while a vertical line is graphed \( x = a \), where \( a \) is the value where the line is.
Simplifying rational expressions

- What is a rational expression? A rational expression is an expression of two polynomials in a ratio, or a fraction. For example; \( \frac{6x+12}{x+2} \) is a rational expression. Both numerator and denominator are polynomials.
- However, all this tells us is we have a vertical asymptote at the value of \( x = -2 \) if we were asked to graph. We want to simplify it down if possible. If we factor a 6 out of the numerator the equation becomes
  \[ \frac{6(x+2)}{x+2} \]
  now we have the number of \( x+2 \) in both numerator and denominator that we can cancel
  \[ \frac{6(x+2)}{x+2} = \frac{6}{1} = 6 \]
- The same rule applies if there is a quadratic in the numerator or denominator. We must first factor the quadratic then find numbers that cancel out in order to simplify.
  \[ \frac{2x-12}{x^2-7x+6} \]
  here we must first factor both top and bottom, then simplify.

*FOR A REVIEW ON FACTORING, ASK FOR ADDITIONAL LESSON

\[
\frac{2x-12}{x^2-7x+6} = \frac{2(x-6)}{(x-6)(x-1)} = \frac{2(\text{factors})}{\text{factors} \times (x-1)} = \frac{2}{x-1}
\]

Simplify each rational expression:
Adding/Subtracting Rational expressions

In order to add any fraction, we must have a common denominator. If the denominators are the same, we can add our fraction straight across. This is also true of a fraction with a variable in the denominator.

\[
\frac{5}{2m} + \frac{4}{2m} = \frac{9}{2m}
\]

\[
\frac{3n + 4}{2n^2 + 5n - 3} - \frac{2n + 1}{2n^2 + 5n - 3} = \frac{3n + 4 - (2n + 1)}{2n^2 + 5n - 3} = \frac{3n + 4 - 2n - 1}{2n^2 + 5n - 3} = \frac{n + 3}{2n^2 + 5n - 3}
\]

Now, factor the quadratic on the bottom using the factoring by grouping method since we have a coefficient that is not the value of 1.

\[
\frac{n + 3}{(2n - 1)(n + 3)} = \frac{n + 3}{(2n - 1)(n + 3)} = \frac{1}{2n - 1}
\]

However, if the denominator is not the same we must first make it the same for both fractions before we can add.

\[
\frac{2}{3x} + \frac{1}{6}
\]
Find the lease common denominator, which will be $6x$. In order to get that in both fractions, we must multiply the first fraction by 2 and the second fraction by $x$. 

$$\frac{2}{3x} + \frac{1}{6} = \frac{2 \cdot 2}{2 \cdot 3x} + \frac{1 \cdot x}{6 \cdot x} = \frac{4}{6x} + \frac{x}{6x}.$$ 

Now we can add or subtract our fraction straight across.

$$\frac{4}{6x} + \frac{x}{6x} = \frac{4 + x}{6x}; \text{we cannot reduce any further}$$

The same is true for more complicated denominators. Multiply the denominator to both top and bottom of the other fraction and it will give you a common denominator that allows you to add the fraction.

$$\frac{5}{c + 2} + \frac{6}{c - 3} = \frac{5(c - 3)}{(c - 3)(c + 2)} + \frac{6(c + 2)}{(c - 3)(c + 2)} = \frac{5c - 15}{(c + 2)(c - 3)} + \frac{6c + 12}{(c + 2)(c - 3)} = \frac{11c - 3}{(c + 2)(c - 3)}$$

ADD OR SUBTRACT

If a common denominator exists, add straight across. If a common denominator does not exist, use multiplication to find one.

13. \( \frac{4}{6t-1} + \frac{3}{6t-1} \)  

14. \( \frac{n}{n+3} + \frac{2}{n+3} \)

15. \( \frac{3y+2}{y+4} - \frac{y-6}{y+4} \)

16. \( \frac{7}{3a} + \frac{2}{5} \)
17. \( \frac{3}{8m^3} + \frac{1}{12m^2} \) 

18. \( \frac{c}{c+5} + \frac{4}{c+3} \) 

19. \( \frac{c^2}{ab} - \frac{a^2}{bc} \) 

20. \( 9 + \frac{x-3}{x+2} \)